1.3 *The Gauss-Seidel Method*

The voltages at each bus can be solved for by using the Gauss-Seidel method. The equation in this case is

$$V_k^{(\alpha)} = \frac{1}{Y_{kk}} \frac{(P_k - jQ_k)}{V_k^{(\alpha-1)^*}} - \frac{1}{Y_{kk}} \left[\sum_{j<k} Y_{kj} V_j^{(\alpha)} + \sum_{j>k} Y_{kj} V_j^{(\alpha-1)} \right] \qquad (1\text{-}13)$$

Voltage at iteration α

Or

$$V_k^{(k+1)} = \frac{\frac{(P_i - jQ_i)}{V_k^{(k)^*}} - \left[\sum_{\substack{j=0 \\ j<i}} Y_{ij} V_j^{(k+1)} + \sum_{j>i} Y_{ij} V_j^{(k)} \right]}{Y_{ii}} \qquad (1\text{-}13)$$

Voltage at iteration (k + 1)

The Gauss-Seidel method was the first **AC** power-flow method to be developed for solution on digital computers. This method is characteristically long in solving due to its slow convergence and often difficulty is experienced with unusual network conditions such as negative reactance branches. The solution procedure is the same as shown in Figure 1-3.

1-3-1 Power Flow Equation

Consider a typical bus of a power system network as shown in Figure 1-7. Transmission lines are represented by their equivalent π models where impedances have been converted to per unit admittances on a common MVA base.

Application of KCL to this bus results in

$$\begin{aligned} I_i &= y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots + y_{in}(V_i - V_n) \\ &= (y_{i0} + y_{i1} + y_{i2} + \cdots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \cdots - y_{in}V_n \end{aligned} \qquad (1\text{-}23)$$

or

$$I_i = V_i \sum_{j=0}^{N} y_{ij} - \sum_{j=1}^{N} y_{ij} V_j \qquad j \neq i \qquad (1\text{-}24)$$

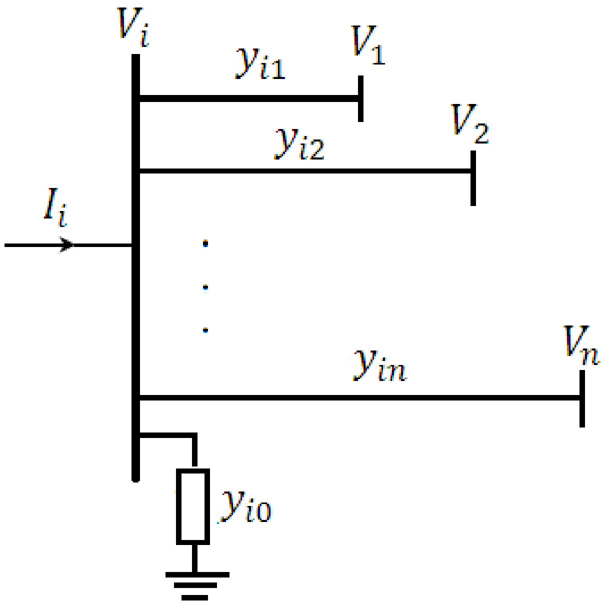

FIGURE 1-7 Atypical bus of the power system

The real and reactive power at bus I is

$P_i + jQ_i = V_i \, I_i^*$	(1-25)

or

$I_i = \dfrac{P_i - jQ_i}{V_i^*}$	(1-26)

Substituting for I_i in (1-24) yields

$\dfrac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{N} y_{ij} - \sum_{j=1}^{N} y_{ij} V_j \qquad j \neq i$	(1-27)

From the above relation, the mathematical formulation of the power flow problem results in system of algebraic nonlinear equations which must be solved by iterative techniques.

1-3-2 Gauss-Seidel Power Flow Solution

In the power flow study, it is necessary to solve the set of nonlinear equations represented by (1-27) for two unknown variables at each node. In the Gauss-Seidel method (1-27) is solved for V_i, and the iterative sequence becomes

$$V_i^{(k+1)} = \frac{\frac{P_i - jQ_i}{V_i^{*(k)}} + \sum_{j=1}^{N} y_{ij} V_j^k}{\sum_{j=0}^{N} y_{ij}} \quad j \neq i \tag{1-28}$$

Where y_{ij} is the actual admittance in per unit, P_i and $Q_i$are the net real and reactive powers expressed in per unit. In writing the KCL, current entering bus i was assumed positive. Thus, for buses where real and reactive powers are injected into the bus, such as generator buses, P_i and Q_i have positive values. For load buses where real and reactive powers are flowing away from the bus, P_i and Q_i have negative values. If (1-27) is solved for $\boldsymbol{P_i}$ **and** $\boldsymbol{Q_i}$, we have

$$P_i^{(k+1)} = Re\{V_i^{*(k)} [V_i^{(k)} \sum_{j=0}^{N} y_{ij} - \sum_{j=1}^{N} y_{ij} V_j^{(k)}]\} \quad j \neq i \tag{1-29}$$

$$Q_i^{(k+1)} = -Im\{V_i^{*(k)} [V_i^{(k)} \sum_{j=0}^{N} y_{ij} - \sum_{j=1}^{N} y_{ij} V_j^{(k)}]\} \; j \neq i \tag{1-30}$$

The power equation is usually expressed in term of the elements of the bus admittance matrix. Since the off-diagonal elements of the bus admittance matrix Y_{bus}, shown by uppercase letters, are $Y_{ij} = -y_{ij}$, and the diagonal elements are $Y_{ii} = \sum y_{ij}$, (1-28) becomes

$$V_i^{(k+1)} = \frac{\frac{P_i - jQ_i}{V_i^{*(k)}} - \sum_{\substack{j=0 \\ j\neq i}}^{N} Y_{ij} V_j^k}{Y_{ii}} \tag{1-31}$$

and

$$P_i^{(k+1)} = Re\left\{ V_i^{*(k)} \left[V_i^{(k)} Y_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{N} Y_{ij} V_j^{(k)} \right] \right\} j \neq i \tag{1-32}$$

$$Q_i^{(k+1)} = -Im\left\{ V_i^{*(k)} \left[V_i^{(k)} Y_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{N} Y_{ij} V_j^{(k)} \right] \right\} j \neq i \tag{1-33}$$

Y_{ii}include the admittance to ground of line charging susceptance and any other fixed admittance to ground.

Since both components of voltage are specified for the slack bus, there are 2(n-1) equations which must be solved by an iterative method. Under normal operating conditions, the voltage magnitude of buses are in the neighborhood of 1.0 per unit or close to the voltage magnitude of the slack bus. Voltage magnitude at load buses are somewhat lower than the slack bus value, depending on the reactive power demand, whereas the scheduled voltage at the generator buses are somewhat higher. Also, the phase angle of the load buses are below the reference angle in accordance to the real power demand, whereas the phase angle of the generator buses may be above the reference value depending on the amount of real power flowing into the bus. Thus, for the Gauss-Seidel method, an initial voltage estimate of 1.0 +j0.0 for unknown voltages is satisfactory, and the converged solution correlates with the actual operating states.

For P-Q buses, the real and reactive powers P_i and Q_i are known. Starting with an initial estimate, (1-31) is solved for the real and imaginary components of voltage. For the voltage-controlled buses (P-V buses) where P_i and $|V_i|$ are specified, first (1-33) is solved for $Q_i^{(k+1)}$, and then is used in (1-31) to solve for $V_i^{(k+1)}$. However, since $|V_i|$is specified, only the imaginary part of $V_i^{(k+1)}$ is retained, and its real part is selected in order to satisfy

$$(e_i^{(k+1)})^2 + (f_i^{(k+1)})^2 = |V_i|^2 \quad (1\text{-}34)$$

$$e_i^{(k+1)} = \sqrt{|V_i|^2 - (f_i^{(k+1)})^2} \quad (1\text{-}35)$$

Where $e_i^{(k+1)}$ and $f_i^{(k+1)}$ are the real and imaginary component of the voltage $V_i^{(k+1)}$in the iterative sequence.

The rate of convergence is increased by applying an acceleration factor to the approximate solution obtained from each iteration.

$$V_i^{(k+1)} = V_i^{(k)} + \alpha(V_{i\,cal}^{(k)} - V_i^{(k)}) \quad (1\text{-}36)$$

where α is the acceleration factor. Its value depends upon the system. The range of 1.3 to 1.7 is found to be satisfactory for typical systems.

The update voltages immediately replace the previous values in solution of the subsequent equations. The process is continued until changes in the real and imaginary components of bus voltages between successive iterations are within a specified accuracy, ie.,

$$|e_i^{(k+1)} - e_i^{(k)}| \leq \varepsilon$$
$$|f_i^{(k+1)} - f_i^{(k)}| \leq \varepsilon \qquad (1\text{-}37)$$

For the power mismatch to be reasonably small and acceptable, a very tight tolerance must be specified on both components of the voltage. A voltage accuracy in the range of 0.00001 to 0.00005 pu is satisfactory. In practice, the method for determining the completion of solution is based on an accuracy index set up on the power mismatch. The iteration continues until the magnitude of the largest element in the ΔP and ΔQ columns is less than the specified value. A typical power mismatch accuracy is 0.001 pu

Once a solution is converged, the net real and reactive powers at the slack bus are computed from (1-32) and (1-33).

1-4 Line Flow and Losses

After the iterative solution of bus voltages, the next step is the computation of line flow and line losses. Consider the line connecting the two buses I and j in Figure (1-8). The line current I_{ij}, measured at bus i and defined positive in the direction

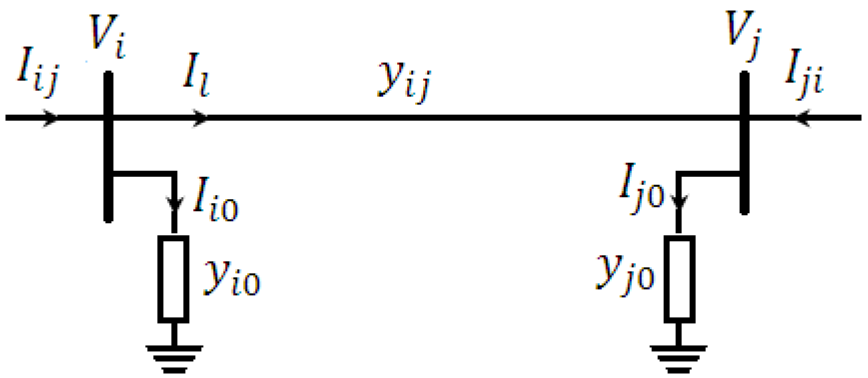

Figure (1-8) Transmission line model for calculation line flow

$i \rightarrow j$ is given by

$I_{ij} = I_l + I_{i0} = y_{ij}(V_i - V_j) + y_{i0}V_i$	(1-38)

Similarly, the line current I_{ji} measured at bus j and defined positive in the direction $\boldsymbol{j} \rightarrow \boldsymbol{i}$ is given by

$I_{ji} = -I_l + I_{j0} = y_{ij}(V_j - V_i) + y_{j0}V_j$	(1-39)

The complex powers S_{ij} from bus i to j and S_{ji} from bus j to i are

$S_{ij} = V_i I_{ij}^*$	(1-40)
$S_{ji} = V_j I_{ji}^*$	(1-41)

The power loss in line $i - j$ is the algebraic sum of the powers flows determined from (1-40) and (1-41), i.e.,

$S_{L\,ij} = S_{ij} + S_{ji}$	(1-42)

The power flow solution by the Gauss-Seidel method is demonstrated in the following example.